An enthalpy balance over the vapor phase up to but not including the interface proper would contain the following terms:

$$\text{Rate of enthalpy in} = G\overline{C}_{NC}(t-t_r) + GH[C_v(t-t_r)+\lambda_r]$$

The rate of enthalpy removal must contain the contribution of the diffusing vapor, which leaves the control volume at a rate $N_A a\, dZ$ (per unit cross section of tower) at a temperature t_i:

$$\text{Rate of enthalpy out} = G\overline{C}_{NC}(t+dt-t_r) + G(H+dH)[C_v(t+dt-t_r)+\lambda_r] + N_A a\, dZ\,[C_v(t_i-t_r)+\lambda_r]$$

The difference in these enthalpy rates is properly attributed to a sensible heat flux from the vapor phase toward the liquid. With Equation (2) there results

$$h'_v a\,(t-t_i)dZ = -Gs\,dt - G\,dH\,C_v\,(t-t_i) \qquad (10)$$

Similarly, an enthalpy balance over the liquid phase up to but not including the interface yields

$$h'_l a\,(t_l-t_i)dZ = LC_l dt_l - G\,dH\,C_l\,(t_i-t_l) \qquad (11)$$

In Equation (11) the contribution of the diffusive flux entering the control volume as a liquid at temperature t_i has been included.

Returning to Equations (3) and (4) one can see that these relations differ from the enthalpy balances just considered in their omission of the right-hand term of Equations (10) and (11). Equations (3) and (4) are therefore approximations of exact enthalpy balances which have been in use so long that they have acquired the aura of fundamental relationships. By their inexact nature they can only lead to anomalies when the internal consistency of the entire set of equations describing the simultaneous heat and mass transfer in the dehumidifier is considered. Their usefulness in the practical task of analyzing this type of contactor depends on the magnitude of the omitted term. The heat transfer coefficients h'_v and h'_l are more nearly equal to those obtainable from correlations of heat transfer experiments conducted in the absence of simultaneous mass transfer [appropriately corrected by the method of Ackermann (2)]. This is generally not true of h_v and h_l.

It can easily be demonstrated that combining Equations (2), (8), (10), and (11) will lead to a "flux balance at the interface" given identically by Equation (5). Nevertheless the flux balance at the interface is still a superfluous concept. The proper enthalpy and material balances are the necessary and sufficient relations for a complete and internally consistent description of the process.

NOTATION

a = interfacial area for both heat and mass transfer per unit volume, sq.ft./cu.ft.
C_l = liquid heat capacity, B.t.u./(lb.)(°F.)
C_v = heat capacity of the condensible vapor, B.t.u./(lb.)(°F.)
$\overline{C}_{NC}$ = average heat capacity of the carrier gas, B.t.u./(lb.)(°F.)
G = mass velocity of carrier gas. lb./(hr.)(sq.ft.)
H = humidity, lb. vapor/lb.carrier gas
h_l, h'_l = liquid-phase heat transfer coefficient defined by Equations (4) and (11) respectively, B.t.u./(hr.)(sq.ft.)(°F.)
h_v, h'_v = gas-phase heat transfer coefficients, defined by Equations (3) and (10) respectively, B.t.u./(hr.)(sq.ft.)(°F.)
k_g = gas-phase mass transfer coefficient, lb. mole/(hr.)(sq.-ft.)(atm.)
L = mass velocity of liquid, lb./(hr.)(sq.ft.)
M_v = molecular weight of the condensible vapor, lb./lb.mole
N_A = rate of mass transfer, lb./(hr.)(sq.ft.)
p, p_i = partial pressure of the condensible vapor in the gas phase and at the interface respectively, atm.
s = humid heat, equal to $\overline{C}_{NC} + HC_v$, B.t.u./(lb.)(°F.)
t = gas-phase temperature, °F.
t_l = liquid-phase temperature, °F.
t_i = temperature of interface, °F.
t_r = reference temperature for enthalpy balance, °F.
Z = height, ft.
λ = heat of vaporization, B.t.u./lb.

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Effectiveness of Mixing Tanks in Smoothing Cyclic Fluctuations

EDGAR B. GUTOFF

Brown Company, Berlin, New Hampshire

Mixing tanks are useful in smoothing out fluctuations in continuous process streams. If perfect mixing is assumed in the mixing tank, calculation of the effectiveness of the tank in reducing cyclic fluctuations can be fairly straightforward, as was shown by Danckwerts and Sellers (*4*), Kramers and Alberda (*8*), Gutoff (*6*), and Walker and Cholette (*13*).

When, on the other hand, the assumption of perfect mixing is not justified, it is extremely difficult to calculate the mixing tank effectiveness, for the fluid regime is very complex. [Danckwerts (*2, 3*) describes how experimental data on a given tank can be used for this calculation.] Cholette and Cloutier (*1*) assumed that in part of the tank volume perfect mixing takes place, that part is stagnant, and that some plug flow and short circuiting of feed occur. However it is difficult to calculate beforehand what the magnitude of these factors will be. The model presented below is believed to be more fundamental in that it takes into account the behavior of the agitator.

Edgar B. Gutoff is with Ionics, Incorporated, Cambridge, Massachusetts.

Either laminar or turbulent flow may take place in the mixing tank. However for longitudinal blending laminar flow is more efficient (9), for there is a greater velocity gradient across the flow path. This velocity gradient causes longitudinal flow and thus promotes the longitudinal blending. The transverse gradients set up by the longitudinal flow are removed by the strong transverse mixing that takes place at the agitator itself. With turbulent flow, on the other hand, the velocity profile is more nearly flat, and there is less back mixing.

If we are conservative, and assume turbulent flow conditions in the mixing tank, and further assume that the turbulent flow can be approximated by a flat velocity profile with no relative longitudinal flow, then the mixing-tank efficiency can be calculated from a simple difference equation. This flow condition may be approximated in practice by a propeller agitator located at the bottom of a draft tube in a tank and also in the case of mixing of certain types of slurries, such as wood pulp. In other cases it is clear that there is a great deal of return of the rising fluid to the agitator at all levels above the propeller, not only at the top. This short-circuiting of the fluid thus aids the longitudinal mixing. The assumed model therefore gives a limiting case, and this case is the conservative limit. Being a conservative limit, the model should prove useful in defining the worst conditions. If short-circuiting is to be taken into account, the mathematics become extremely involved and so far have not been solved.

A turbine agitator provides a radically different flow pattern in the tank. The fluid flows radially outward at the agitator, up and down the sides to the top and bottom of the tank respectively, and then along the axis of the tank back to the turbine. A tank with a turbine agitator is thus similar to two tanks like those described above, but each would be half the height and they would be placed bottom to bottom. Although the back mixing of the fluid is greater in this case, the assumption of a flat velocity profile will lead to the identical conservative results obtained with the first model. Figure 1 is equivalent to the above models, but here the agitation takes place by recycling some of the product back to the input stream. The mathematics below refers to Figure 1.

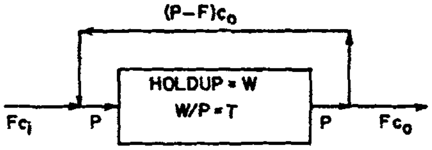

Fig. 1. Equivalent model of a continuous mixer, with throughput rate *F* and agitator turnover time *T*.

The material at the entrance point at t is a mixture of the input, $Fc_i(t)$, and the recycle, $(P-F)c_o(t)$. When one assumes a flat velocity profile, all of this takes time T to move through the system, to give the output flow just before the take-off point of $Pc_o(t+T)$ at time $t+T$. Thus

$$c_o\,(t+T) = \frac{F}{P}\,c_i\,(t) + \frac{(P-F)}{P}\,c_o(t) \qquad (1)$$

This is a difference equation and may be solved by standard methods (7).

The steady state solution may be found by the method of undetermined coefficients (7). If the input function is assumed to vary sinusoidally,

$$c_i(t) = \bar{c}\,(1 + b \sin 2\pi t/\lambda) \qquad (2)$$

then the output function will be of the form (7)

$$c_o(t) = A + B \sin 2\pi t/\lambda + C \cos 2\pi t/\lambda \qquad (3)$$

and

$$c_o(t+T) = A + B \sin 2\pi\,(t+T)/\lambda + C \cos 2\pi\,(t+T)/\lambda \qquad (4)$$

Substituting Equations (2), (3), and (4) into Equation (1) and equating constant coefficients, coefficients of sin $2\pi t/\lambda$, and coefficients of cos $2\pi t/\lambda$ gives three simultaneous equations involving A, B, and C. Solving these equations and simplifying yields the desired result,

$$\frac{c_o - \bar{c}}{bc} = \frac{-\left[1 - \dfrac{1}{H/T} - \cos 2\pi T/\lambda\right] \sin 2\pi t/\lambda}{2\left(\dfrac{H}{T} - 1\right)\left[1 + \dfrac{1}{2\dfrac{H}{T}\left(\dfrac{H}{T} - 1\right)} - \cos 2\pi T/\lambda\right]} - \frac{\sin 2\pi T/\lambda\ \cos 2\pi t/\lambda}{2\left(\dfrac{H}{T} - 1\right)\left[1 + \dfrac{1}{2\dfrac{H}{T}\left(\dfrac{H}{T} - 1\right)} - \cos 2\pi T/\lambda\right]} \qquad (5)$$

The effectiveness of the mixing tank in smoothing the input fluctuations may be expressed most conveniently by the reduction in the root mean square value of the deviations from the mean (4). This is the standard deviation σ. When

$$c - \bar{c} = a \sin\theta + b \cos\theta \qquad (6)$$

then

$$\sigma = \sqrt{(a^2 + b^2)/2} \qquad (7)$$

Therefore from Equation (5)

$$\sigma_o = \frac{b\bar{c}}{\sqrt{2}}\,\frac{1}{\left[1 + \dfrac{2H}{T}\left(\dfrac{H}{T} - 1\right)(1 - \cos 2\pi T/\lambda)\right]^{1/2}} \qquad (8)$$

and for the input, from Equation (2)

$$\sigma_i = \frac{b\bar{c}}{\sqrt{2}} \qquad (9)$$

Therefore the effectiveness of the mixing tank is found as

$$1 - E = \frac{\sigma_o}{\sigma_i} = \frac{1}{\left[1 + \dfrac{2H}{T}\left(\dfrac{H}{T} - 1\right)(1 - \cos 2\pi T/\lambda)\right]^{1/2}} \qquad (10)$$

When the turnover time approaches zero, perfect mixing is approached; the cosine function can be replaced by the first terms in its series expansion:

$$1 - \cos 2\pi T/\lambda \approx (2\pi T/\lambda)^2/2 \qquad (11)$$

$(H/T - 1)$ approaches H/T, and Equation (10) reduces to

$$1 - E^\circ = \frac{\sigma^\circ_o}{\sigma_i} = \frac{1}{[1 + (2\pi H/\lambda)^2]^{1/2}} \qquad (12)$$

This is the equation for the effectiveness of a mixing tank with perfect mixing (4), such that the concentration in the tank is at all times equal to the output concentration.

The mixing-tank effectiveness calculated from Equation (10) is plotted as a function of H/λ, the ratio of holdup time to the cycle time of the fluctuations, for various values of H/T, the ratio of holdup time to turnover time,

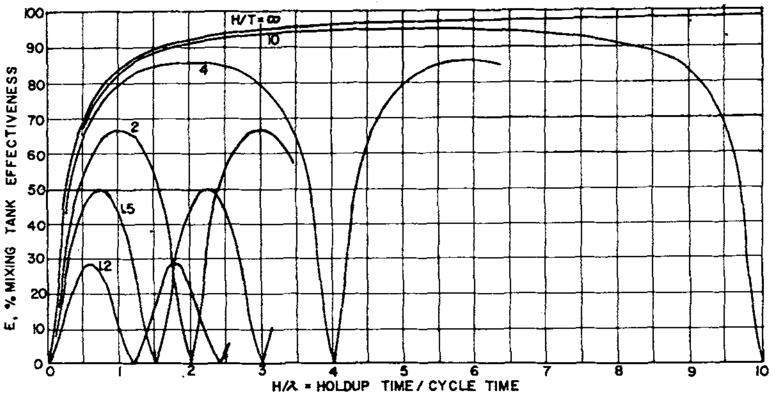

Fig. 2. Effectiveness of mixing tanks in reducing cyclic fluctuations as a function of the ratio of tank holdup time to the cycle time of the fluctuations. H/T is the ratio of the holdup time to the agitator turnover time.

in Figure 2. The ratio of H/T is also equal to P/F, the ratio of agitator pumping rate to throughput rate. It is seen that the effectiveness increases as the turnover time decreases relative to the holdup time, or as perfect mixing is approached, and also increases as the ratio of holdup time to cycle time increases.

As the ratio of holdup time to cycle time increases, the effectiveness goes through the maxima and then decreases to zero at integral values of T/λ, equal to $H/\lambda \div H/T$. This occurs only because the proposed model assumes a flat velocity profile, with no short-circuiting back to the agitator, and the flow in the mixer is in phase with the frequency of the fluctuations. In an actual mixing tank it is extremely doubtful whether these maxima would appear, because of the back mixing that takes place. Certainly the effectiveness would not drop to zero at values of $T/\lambda = 1$, 2, and higher integers.

EFFECTIVENESS OF AGITATION

The effectiveness of agitation measuring the approach to perfect mixing, may be defined as the mixing-tank effectiveness divided by the tank effectiveness if perfect mixing took place (5), E° or

$$E = E_a E^\circ \tag{13}$$

The effectiveness of agitation is plotted in Figure 3 as a function of H/T, the ratio of the holdup time to the turnover time, for various values of T/λ. It is noticed that the lines of constant T/λ are asymmetrical with respect to $T/\lambda = 0.5$. Looking again at Figure 2, one notices that although the curves of tank effectiveness are symmetrical (about the point $T/\lambda = 0.5$), the value of tank effectiveness with perfect mixing keeps rising with increasing values of H/λ, and so the degrees of agitation are lower for values of T/λ above 0.5 than for those under 0.5.

The effectiveness of agitation, as defined above, is thus dependent on the cycle time of the fluctuations. Whether this dependence exists in practice or is only a result of using an approximate model is not known. Certainly, the dependence would not be so great as shown in Figure 3.

The effectiveness of agitation is above 90% in most cases when the turnover time is less than one fifth of the holdup time (H/T above 5).

AGITATOR PUMPING RATES

So far no mention has been made here of the determination of agitator pumping rates. The subject is complex; some work has been done, and more needs to be done. Rushton and coworkers (9, 10) presented some equations to calculate this pumping rate, for both propellers (9) and turbines (10). They pointed out that the total fluid pumped was greater than the discharge from the agitator and discussed (9) the application of the work of Folsom and Ferguson (5) on jet entrainment to this problem. Van de Vusse (12) derived equations enabling one to calculate the pumping rates for turbines, paddle stirrers, and propellers. It is hoped that this paper will stimulate more studies in this field.

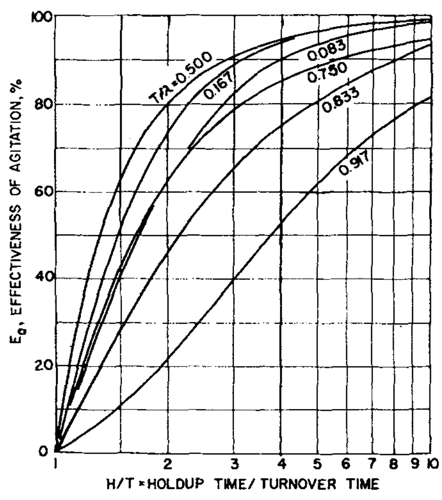

Fig. 3. Effectiveness of agitation in continuous mixing tanks as a function of the ratio of tank holdup time to agitator turnover time. T/λ is the ratio of turnover time to the cycle time of fluctuations. The effectiveness of agitation measures the approach to perfect mixing.

NOTATION

A, a = constants
B, b = constants
b = maximum fractional deviation of c_i from $\bar{c}$, or $[(c_i - \bar{c})/\bar{c}]_{max}$
C = constant
c_i = input concentration or other additive intensive property
c_o = output concentration
$\bar{c}$ = time average concentration, in and out
E = mixing-tank effectiveness, equal to $1-(\sigma_o/\sigma_i)$
E° = mixing-tank effectiveness with perfect mixing
E_a = degree of agitation, equal to E°/E
F = flow rate in and out of mixing tanks, lb./min.
H = holdup time in mixing tank, equal to W/F, min.
P = agitator pumping rate, lb./min.
T = turnover time in mixing tank, equal to W/P, min.
t = time, min.
W = holdup in mixing tank, lb.
λ = cycle time of fluctuations, min.
σ_i = standard deviation of input fluctuations
σ_o = standard deviations of output fluctuations
σ°_o = standard deviations of output fluctuations with perfect mixing

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NUCLEAR ENGINEERING—PART VI, Vol. 55, No. 23, 1959.

A Digital Program to Evaluate Transients for Nuclear Power Plant Design, F. J. Scheib and A. J. Arker. The design limitations of a nuclear power plant for mobile use often depend on the transients which the plant undergoes during normal or casualty operation. Examples of decisions which depend on transient evaluation in the design of a pressurized water plant are given, and the model digital computer program used to analyze such transients is described. Important features of this program are nonsymmetrical and variable flow operation of the two coolant loops, use of nonanalytic input data, and flexibility for alteration. **Control of the PRTR-A Gas-Balance Method With Supplementary Mechanical Shims,** J. F. Fletcher. The Plutonium Recycle Program, a comprehensive research and development program, has as its goal the development of safe, economical methods of using plutonium fuels in power reactors. The Plutonium Recycle Test Reactor is a test facility essential to this program designed to have a high degree of versatility and to be capable of operating under a great variety of experimental conditions. The control system used provides an unusual degree of reactor safety. **A Study of the Feasibility of a Tracer System for Locating a Fuel-Element Failure in a Pressurized Water Reactor,** Meyer Pobereskin, Duane N. Sunderman, Aaron Eldridge, George D. Calkins, and Walston Chubb. This investigation indicated that a system of tracer elements for location of fuel-element failure merited development. The method consists in adding small amounts of different pairs of elements to the fuel alloy of each subassembly. When a fuel-element failure occurs, a portion of the coolant is analyzed radiochemically for the elements added to the fuel as tracers. The radioactive species of the elements detected indicate the subassembly in which the failure has occurred. Eleven elements were found to be suitable tracers. **Coolant Temperature Rise in a Pressurized Water Reactor During a Loss of Flow Incident,** Joel Weisman, Shepard Bartnoff, and G. C. Tirellis. A method is described for calculating the coolant temperature at the reactor outlet as a function of time following a loss of pumping power and the consequent loss of coolant flow. The effects of negative temperature coefficient and delayed gamma radiation on the heat production rate of the reactor are considered and contributions to the total coolant temperature rise calculated from both the nucleate boiling region and the nonnucleate boiling region of the core. **Reprocessing Costs for Fuel From a Single-Region Aqueous Homogeneous Reactor,** A. C. Jealous and R. J. Klotzbach. The cost is estimated for a fuel-reprocessing plant to recover fissionable, fertile, and moderator materials from irradiated slurry fuel of the Pennsylvania Advanced Reactor (PAR). The estimate is based on a reprocessing plant designed to process 47 kg. of thorium per day for return to the reactor plant. Off-site solvent-extraction decontamination of PAR fuel in A.E.C. multipurpose plant would cost more than complete-on-site reprocessing. **Pilot Plant Fluorination of Uranium Fuel Elements by Bromine Trifluoride,** Gerald Strickland, F. L. Horn, and Richard Johnson. The dissolution of uranium in bromine trifluoride at 250°F. was investigated in continuous equipment on a pilot plant scale. In particular the autocatalytic effect of the UF_6 concentration on the dissolution time of unirradiated B.N.L. reactor slugs was determined. A series of nine runs is described in which the UF_6 concentration progressed from 0 to 4.5 mole %. **Reprocessing Uranium-Zirconium-Alloy Reactor Fuel Elements,** C. B. Leek, R. B. Lemon, and F. K. Wrigley. The process employed for the recovery of uranium from uranium-zirconiumalloy reactor fuel ele-